Intermediate Microeconomics

Chapter 5: Choice

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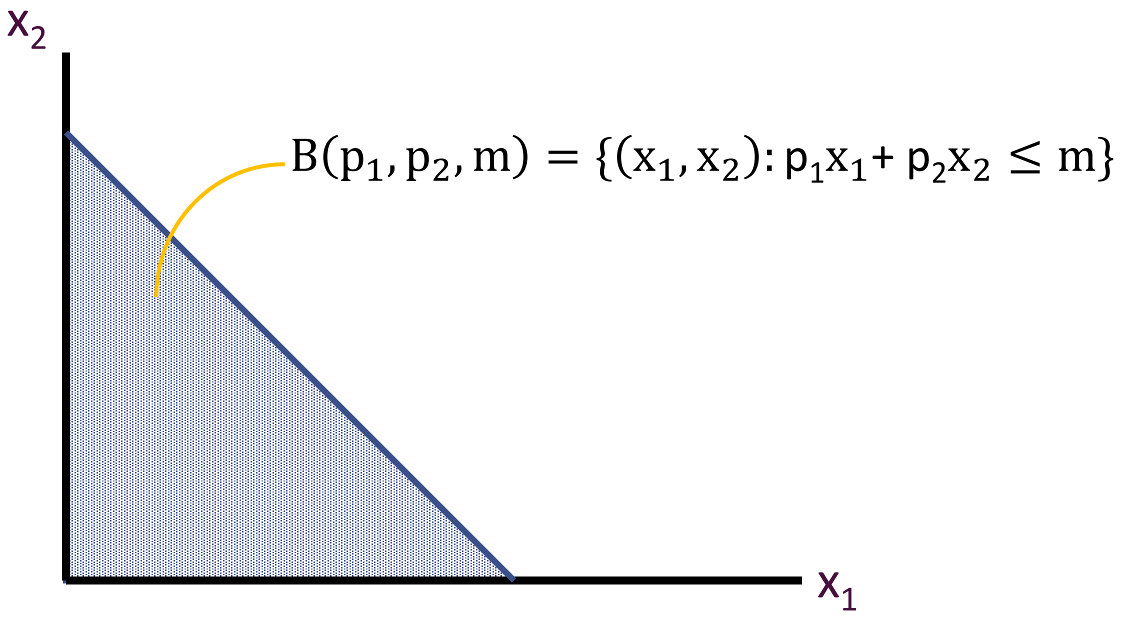
Economic Model of Consumer Choice

Economists assume that consumers choose the best bundle of goods they can afford.

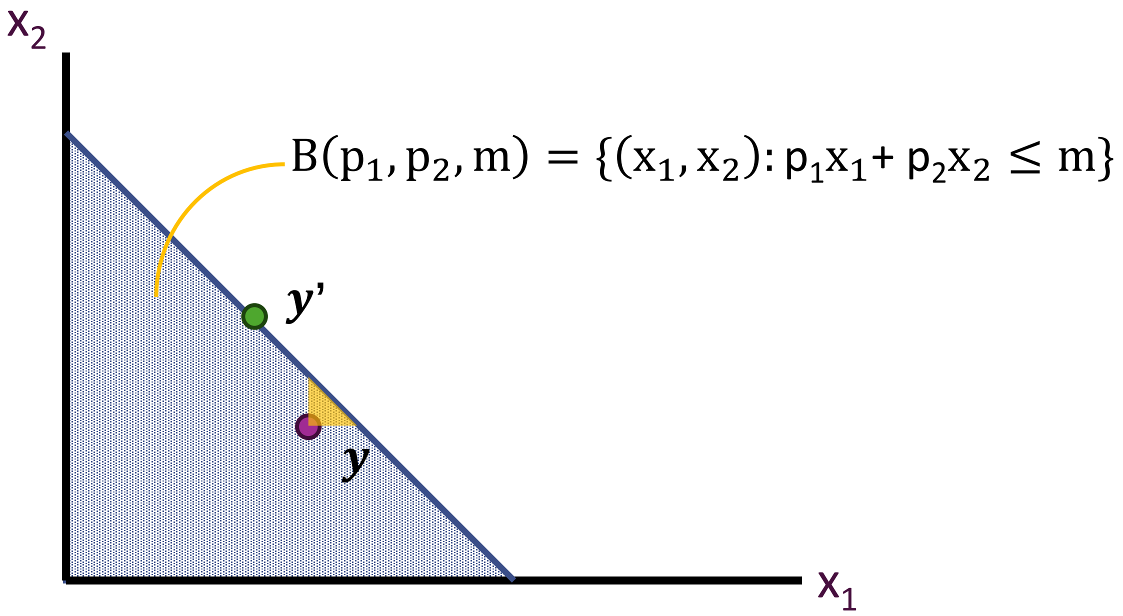
Consumers choose the most preferred bundle from their budget sets.

Utility-Maximization Process



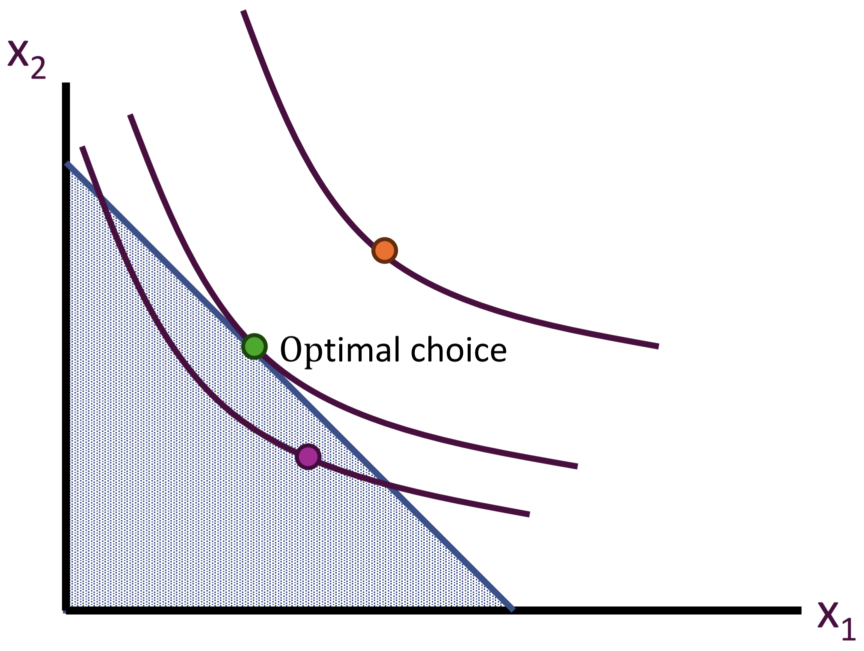
Utility-Maximization Process (cont'd)

Walras' Law :
The consumer's choice will always lie on the boundary of the budget line, rather than within the interior of the budget constraint set.



Walras' Law

Utility-Maximization Process (cont'd)



We can add the individual's utility map to show the utility-maximization process:

— The individual can do better than purple point by reallocating his budget

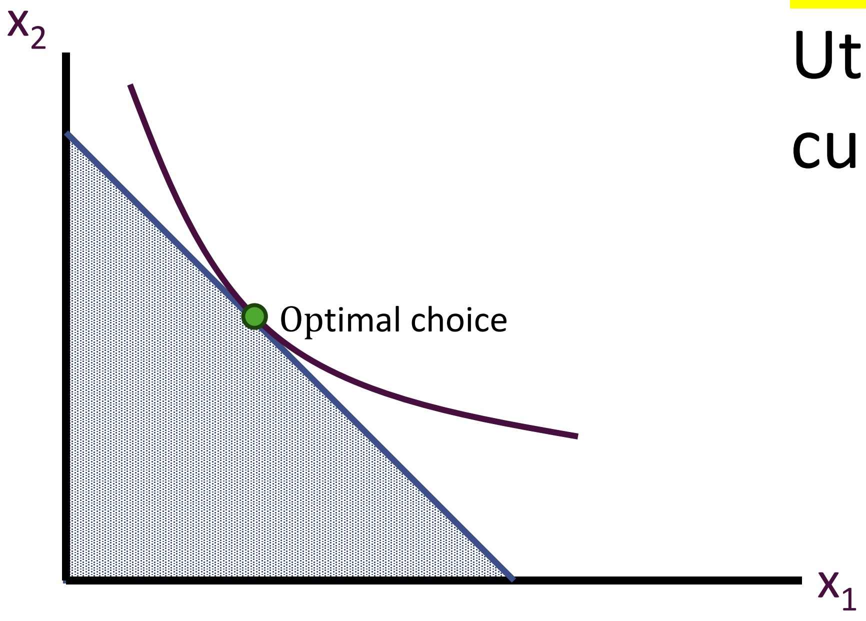
— The individual cannot have orange point because income is not large enough

— Green point is the point of utility maximization

FOC for a Utility-Maximization

$$\max_{(x_1, x_2)} u(x_1, x_2)$$

$$\text{s.t. } (x_1, x_2) \in B(p_1, p_2, m) \Rightarrow p_1x_1 + p_2x_2 = m$$



First-Order Conditions for a Maximum：

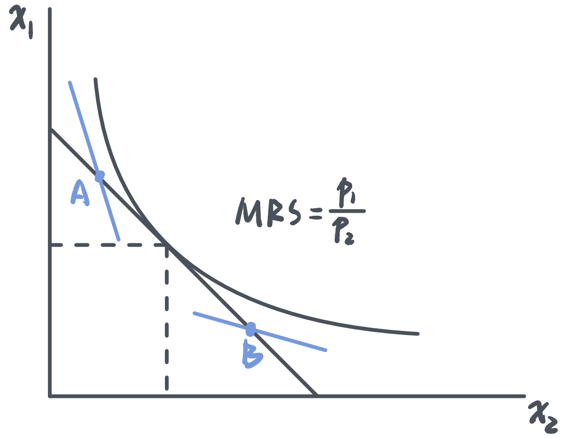
Utility is maximized where the indifference curve is tangent to the budget constraint

$$MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}$$

$$\Rightarrow \frac{MU_1}{MU_2} = \frac{p_1}{p_2}$$

$$\Rightarrow \frac{MU_1}{P_1} = \frac{MU_2}{p_2}$$

FOC for a Utility-Maximization



Suppose MRS$\neq \frac{p_1}{p_2}$ (not tangent)

A: (1) If MRS $> \frac{\mathrm{p}_1}{\mathrm{p}_2}$:

—It is steeper than budget

—Consumer are willing to give up x_2 for x_1 than market requires

—Can increase utility by lowering x_2 while increasing x_1

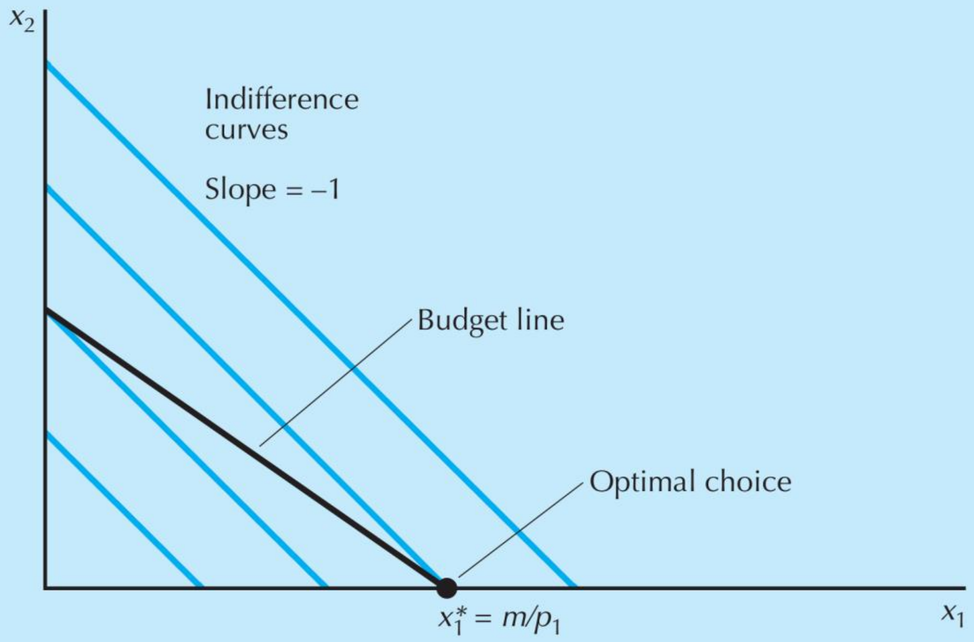
B: (2) If MRS $> \frac{\mathrm{p}_1}{\mathrm{p}_2}$:

—It is flatter than budget

—Consumer are less willing to trade x_2 for x_1 than market requires

—Can increase utility by lowering x_1 while increasing x_2

Does Tangent Condition Always Work?



Optimal choice with perfect substitutes. If the goods are perfect substitutes, the optimal choice will usually be on the boundary.

maximum ⇄ tangent condition 必要条件.

Does Tangent Condition Always Work?

No

Corner solution: Optimal solution (x_1^*, x_2^*) such that either $(x_1^* > 0, x_2^* = 0)$ or $(x_1^* = 0, x_2^* > 0)$

i.e., Consumers spend all income on one good

In this case, MRS$\neq \frac{p_1}{p_2}$

Utility Maximization: Summary

Optimal choice problem is to

(1) choose a consumption bundle (x_1, x_2)

(2) maximize utility $u(x_1, x_2)$

(3) subject to $p_1x_1 + p_2x_2 = m$, $x_1 \geq 0$, and $x_2 \geq 0$

We start by solving for interior solution with MRS$= \frac{p_1}{p_2}$

If no solutions, or if we find $x_i < 0$ (negative quantity), then assume optimal solution will not have tangency (i.e., corner solution).

Utility Maximization: Example 1 (Cobb-Douglas)

Suppose given $u(x_1, x_2) = x_1^{\gamma} x_2^{1-\gamma}$ $\gamma \in (0, 1)$.

Set up:

$$\max u(x_1, x_2) = x_1^{\gamma} x_2^{1-\gamma}$$
$$\text{s.t.,} \quad x_1 \geq 0, \quad x_2 \geq 0$$
$$p_1 x_1 + p_2 x_2 = m$$

$$MRS = \frac{MU_1}{MU_2} = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \frac{\gamma x_1^{\gamma-1} x_2^{1-\gamma}}{(1-\gamma) x_1^{\gamma} x_2^{-\gamma}} = \frac{\gamma x_2}{(1-\gamma) x_1} = \frac{p_1}{p_2}$$

$$p_1 x_1 + p_2 x_2 = m \qquad x_2 = \frac{p_1 (1-\gamma) x_1}{\gamma p_2}$$

Solve the utility-maximization problem through MRS$= \frac{p_1}{p_2}$ and get demand functions:

$$\left(p_1 + \frac{1-\gamma}{\gamma} p_1\right) x_1 = m$$

$$x_1^* = ?$$
$$x_2^* = ?$$

$$\begin{cases} x_1^* = \frac{\gamma m}{p_1} \\ x_2^* = \frac{(1-\gamma) m}{p_2} \end{cases}$$

Utility Maximization: Example 1 (Cobb-Douglas)

Suppose given $u(x_1, x_2) = x_1^{\gamma} x_2^{1-\gamma}$ $\gamma \in (0, 1)$.

Set up:

$$\max u(x_1, x_2) = x_1^{\gamma} x_2^{1-\gamma}$$
$$\text{s.t.}, \quad x_1 \geq 0, \quad x_2 \geq 0$$
$$p_1 x_1 + p_2 x_2 = m$$

Solve the utility-maximization problem through MRS$= \frac{p_1}{p_2}$ and get demand functions:

$$x_1^* = \gamma m / p_1$$
$$x_2^* = (1-\gamma) m / p_2$$

Utility Maximization: Example 1 (Cobb-Douglas)

Solve the utility-maximization problem through the use of Lagrange multipliers (λ).

Set up the Lagrangian

$$L = u(x_1, x_2) - \lambda(p_1 x_1 + p_2 x_2 - m)$$

and differentiate to get the three first order conditions

$$\frac{\partial L}{\partial x_1} = \frac{\partial u(x_1^*, x_2^*)}{\partial x_1} - \lambda p_1 = 0$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial u(x_1^*, x_2^*)}{\partial x_2} - \lambda p_2 = 0$$

$$\frac{\partial L}{\partial \lambda} = p_1 x_1^* + p_2 x_2^* - m = 0$$

Utility Maximization: Example 2 (Perfect Complements)

Suppose given $u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$ $\alpha > 0, \beta > 0$

Set up:

$$\max u(x_1, x_2) = \min(\alpha x_1, \beta x_2)$$

$$\text{s.t.,} \quad x_1 \geq 0, \quad x_2 \geq 0$$

$$p_1 x_1 + p_2 x_2 = m$$

$$\begin{cases} \alpha x_1 = \beta x_2 \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

$$(p_1 + p_2 \frac{\alpha}{\beta}) x_1 = m$$

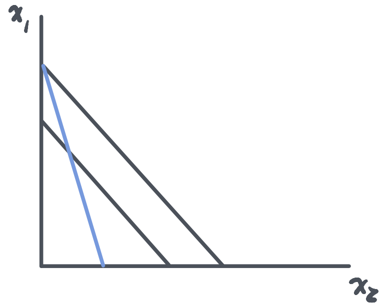
$$\begin{cases} x_1^* = \frac{\beta m}{\beta p_1 + \alpha p_2} \\ x_1^* = \frac{\alpha m}{\beta p_1 + \alpha p_2} \end{cases}$$

Solve the utility-maximization problem and get demand functions:

x_1^*=? $\quad x_1^* = \frac{\beta m}{\beta p_1 + \alpha p_2}$

x_2^*=? $\quad x_2^* = \frac{\alpha m}{\beta p_1 + \alpha p_2}$

Utility Maximization: Example 3 (Perfect Substitutes)



Suppose given $u(x_1, x_2) = \alpha x_1 + \beta x_2$ $\quad \alpha > 0, \beta > 0$

$MRS = \frac{\partial u / \partial x_1}{\partial u / \partial x_2} = \frac{\alpha}{\beta} = \frac{p_1}{p_2}$.

if $MRS > \frac{p_1}{p_2}$, $(x_1^*, x_2^*) \to (\frac{m}{p_1}, 0)$

$MRS < \frac{p_1}{p_2}$, $(x_1^*, x_2^*) \to (0, \frac{m}{p_2})$.

Set up:

$$\max u(x_1, x_2) = \alpha x_1 + \beta x_2$$

$$\text{s.t.}, \quad x_1 \geq 0, \quad x_2 \geq 0$$

$$p_1 x_1 + p_2 x_2 = m$$

Solve the utility-maximization problem and get demand functions:

$$x_1^* = ?$$

$$x_2^* = ?$$

Utility Maximization: Example 4 (n-Good Case)

Suppose given $u(x_1, x_2, \ldots, xn)$

Set up:

$$\max u(x_1, x_2, \ldots, xn)$$

$$\text{s.t.,} \quad x_1 \geq 0, \quad x_2 \geq 0, \ldots, x_n \geq 0$$

$$p_1 x_1 + p_2 x_2 + \cdots + pn x_n = m$$

Solve the utility-maximization problem and get demand functions

Constant Elasticity of Substitution
恒定替代彈性

Utility Maximization: Example 5 (CES)

Suppose given $u(x_1, x_2) = (\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}}$

$$\frac{\partial u}{\partial x_1} = \frac{1}{\rho}\left(\frac{\partial}{\partial x_1}(\alpha x_1^{\rho} + \beta x_2^{\rho})\right)^{\frac{1}{\rho}-1} \cdot (\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}-1}$$

$$= \frac{1}{\rho}(\alpha \rho x_1^{\rho-1})(\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}-1}$$

$$= \alpha x_1^{\rho-1}(\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1-\rho}{\rho}}.$$

$$\frac{\partial u}{\partial x_2} = \beta x_2^{\rho-1}(\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1-\rho}{\rho}}.$$

Set up:

$$\max\ u(x_1, x_2) = (\alpha x_1^{\rho} + \beta x_2^{\rho})^{\frac{1}{\rho}}$$
$$\text{s.t.},\quad x_1 \geq 0,\quad x_2 \geq 0$$
$$p_1 x_1 + p_2 x_2 = m$$

$$\begin{cases} MRS = \dfrac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = \dfrac{\alpha x_1^{\rho-1}}{\beta x_2^{\rho-1}} = \dfrac{p_1}{p_2}. \\ p_1 x_1 + p_2 x_2 = m \end{cases}$$

$\Rightarrow$.

Solve the utility-maximization problem and get demand functions:

$$x_1^* = ?$$
$$x_2^* = ?$$

Intermediate Microeconomics

Chapter 6: Demand

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Demand Functions

Set up:

$$\max_{(x_1, x_2)} u(x_1, x_2)$$

$$\text{s.t. } p_1 x_1 + p_2 x_2 \leq m$$

We write the demand functions as

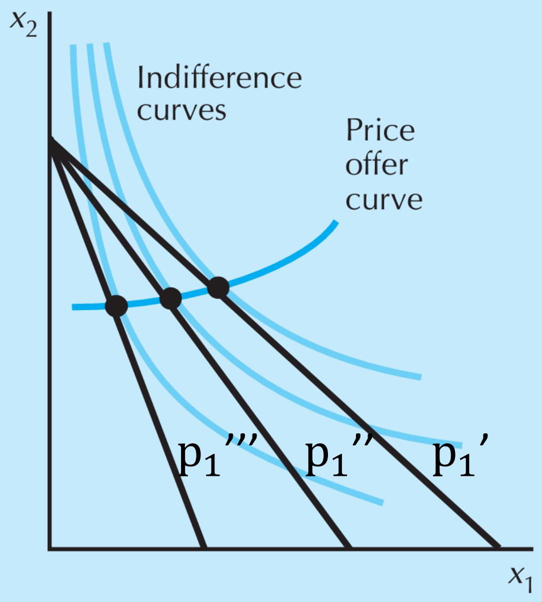
$$x_1 = x_1(p_1, p_2, m)$$
$$x_2 = x_2(p_1, p_2, m)$$

Own-Price Changes

How does ${x_1}^*(p_1, p_2, m)$ change as p_1 changes, holding p_2 and m constant?

Suppose only p_1 increases, from p_1’ to p_1’’ and then to p_1’’’.

Own-Price Changes



A Price offer curve

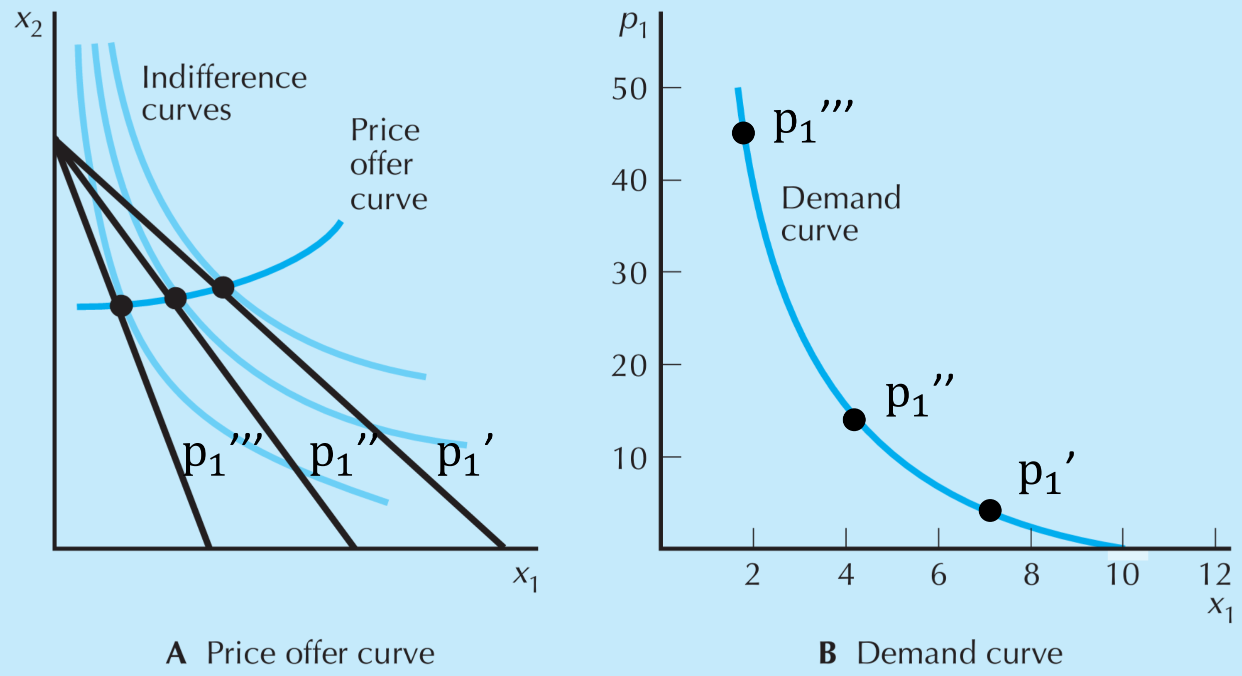
The price offer curve and demand curve. Panel A contains a price offer curve, which depicts the optimal choices as the price of good 1 changes.

价格提供线（⇒需求曲线）

Price offer curve:

—connect together the optimal points

Own-Price Changes



The price offer curve and demand curve. Panel A contains a price offer curve, which depicts the optimal choices as the price of good 1 changes. Panel B contains the associated demand curve, which depicts a plot of the optimal choice of good 1 as a function of its price.

Price offer curve:

—connect together the optimal points

—represents the bundles that would be demanded at different prices for good 1.

—depict in a different way: demand curve

Price Effects

自身价格效应

Own-price effect ($\frac{\partial x_1(p_1,p_2,m)}{\partial p_1}, \frac{\partial x_2(p_1,p_2,m)}{\partial p_2}$):

— In most cases, the own price effect is negative ($\frac{\partial x_1}{\partial p_1} < 0$), meaning there is an inverse relationship between price and quantity demanded (Law of Demand).

— Giffen goods: , the own price effect is positive ($\frac{\partial x_1}{\partial p_1} > 0$).

交叉价格效应

Cross-price effect ($\frac{\partial x_1(p_1,p_2,m)}{\partial p_2}, \frac{\partial x_2(p_1,p_2,m)}{\partial p_1}$):

— Substitutes: $\frac{\partial x_1}{\partial p_2} > 0$ $p_2 \uparrow, x_2 \downarrow, x_1 \uparrow$ (substitute →)

— Complements: $\frac{\partial x_1}{\partial p_2} < 0$ $p_2 \uparrow, x_2 \downarrow, x_1 \downarrow$. (complement →)

— Unrelated: $\frac{\partial x_1}{\partial p_2} = 0$

Own-Price Changes : Cobb-Douglas

What does a p_1 price-offer curve look like for Cobb-Douglas utility function?

Take

$$u(x_1, x_2)= x_1^{\alpha} x_2^{\beta} \quad \text{s.t., } p_1x_1+ p_2x_2=m$$

Then the ordinary demand functions for good 1 and good 2 are

$$x_1^*(p_1, p_2, m)=?$$

and

$$x_2^*(p_1, p_2, m)=?$$

Own-Price Changes: Cobb-Douglas

What does a p_1 price-offer curve look like for Cobb-Douglas utility function?

Take

$$u(x_1, x_2) = x_1^{\alpha} x_2^{\beta} \quad \text{s.t., } p_1x_1 + p_2x_2 = m$$

Then the ordinary demand functions for good 1 and good 2 are

$$x_1^*(p_1, p_2, m) = \frac{\alpha}{\alpha+\beta}\frac{m}{p_1}$$

$\frac{\partial x_1}{\partial p_2} = 0$, $\frac{\partial x_2}{\partial p_1} = 0$

无关商品.

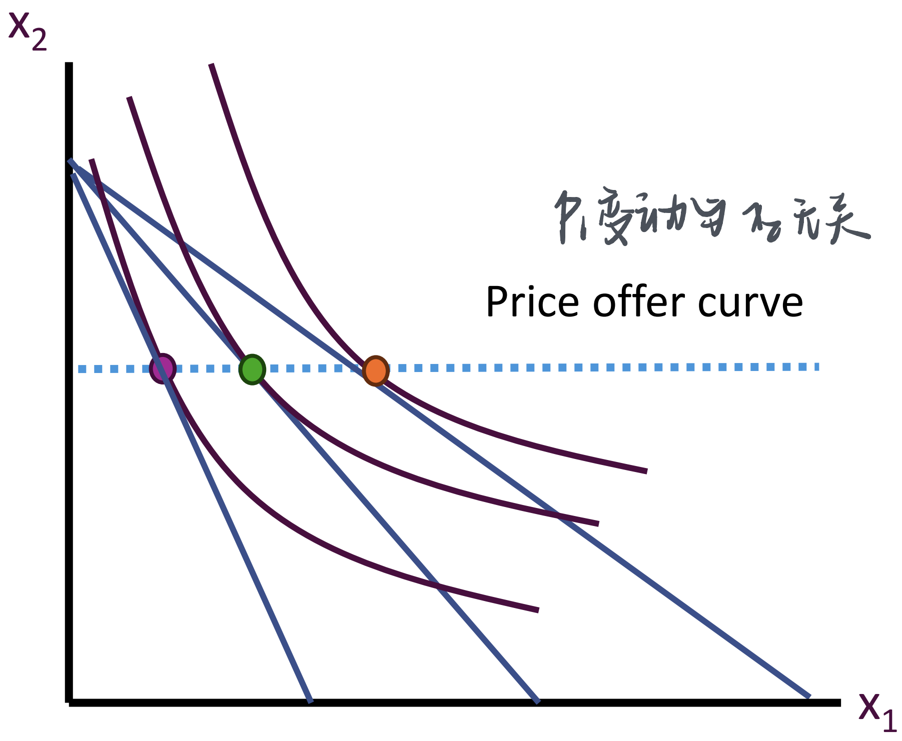
and

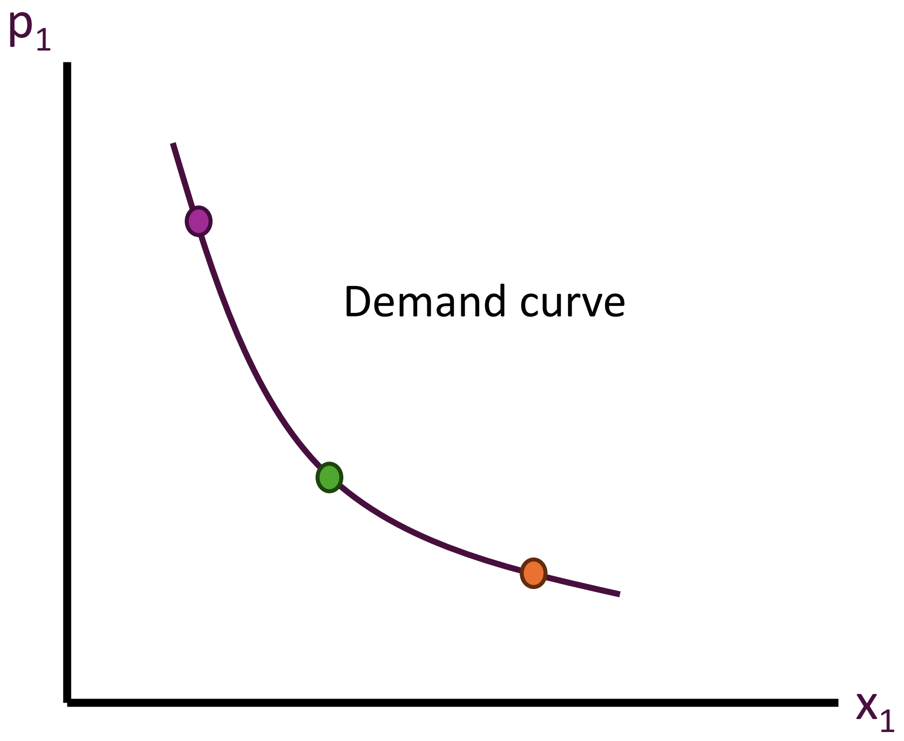
$$x_2^*(p_1, p_2, m) = \frac{\beta}{\alpha+\beta}\frac{m}{p_2}$$

Own-Price Changes: Cobb-Douglas

What does a p_1 price-offer curve look like for Cobb-Douglas utility function?

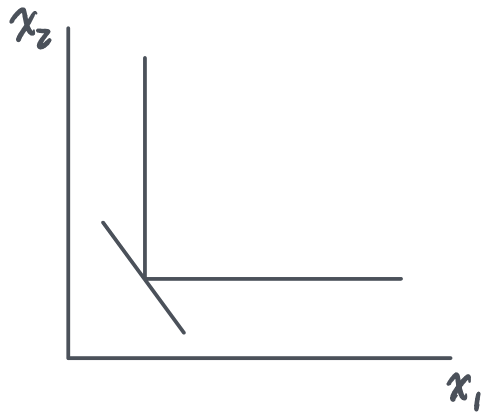
(p_1 changes, holding p_2 and m constant)





Own-Price Changes : Perfect Complements

What does a p_1 price-offer curve look like for perfect-complements (Leontief) utility function?



Take

$$u(x_1, x_2)=\min(x_1, x_2)$$

Then the ordinary demand functions for good 1 and good 2 are

$$x_1^*(p_1, p_2, m)=x_2^*(p_1, p_2, m)=\frac{m}{p_1 + p_2}$$

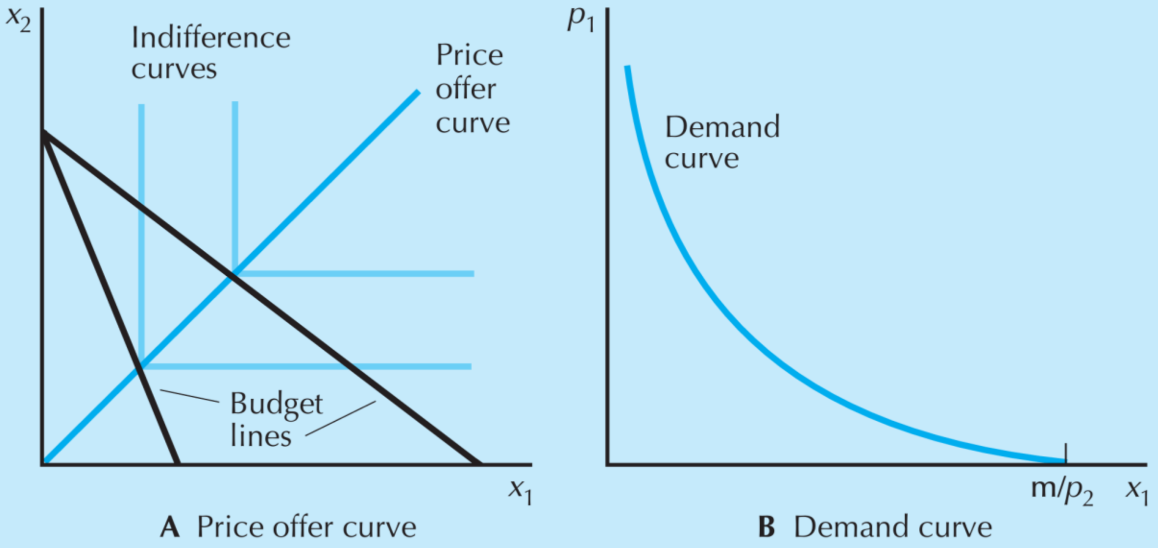
相当于一个商品,

Own-Price Changes : Perfect Complements

What does a p_1 price-offer curve look like for perfect-complements (Leontief) utility function?

$p_1 \to 0$, $x_1 = x_2 = \frac{m}{p_2}$.

$p_1 \to \infty$, $x_1 = x_2 = 0$.



Perfect complements. Price offer curve (A) and demand curve (B) in the case of perfect complements.

Own-Price Changes : Perfect Substitutes

What does a p_1 price-offer curve look like for perfect-substitutes utility function?

Take

$$u(x_1, x_2) = x_1 + x_2$$

Then the ordinary demand functions for good 1 and good 2 are

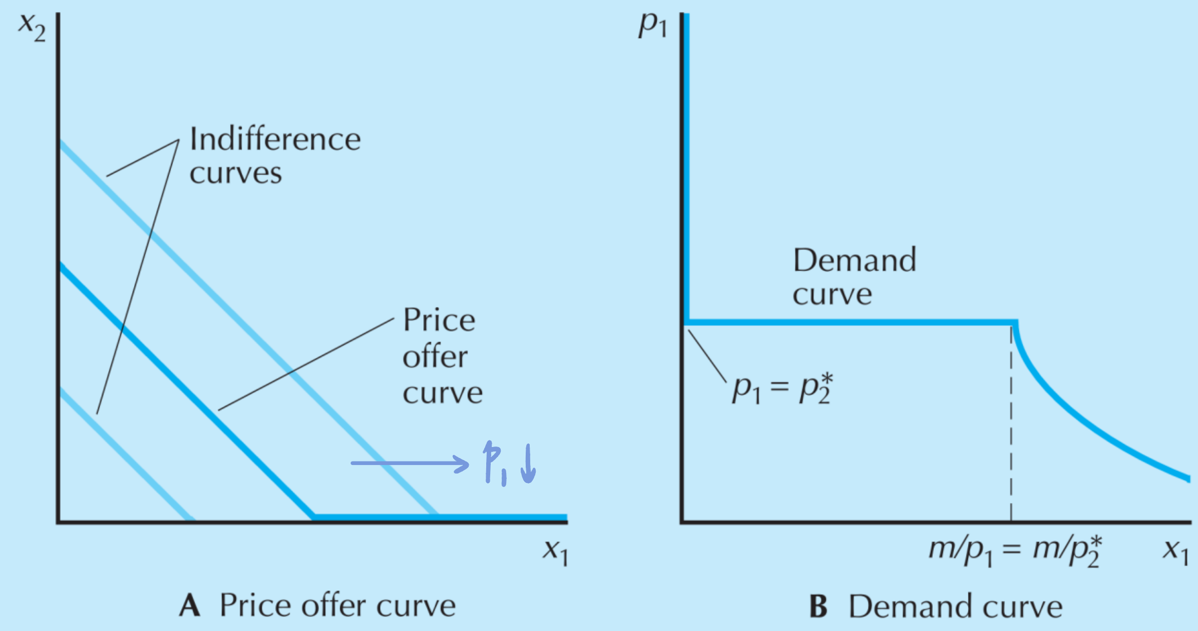
$$x_1^*(p_1, p_2, m) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ m/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

and

$$x_2^*(p_1, p_2, m) = \begin{cases} 0 & , \text{if } p_2 > p_1 \\ m/p_2 & , \text{if } p_2 < p_1 \end{cases}$$

Own-Price Changes : Perfect Substitutes

What does a p_1 price-offer curve look like for perfect-substitutes utility function?



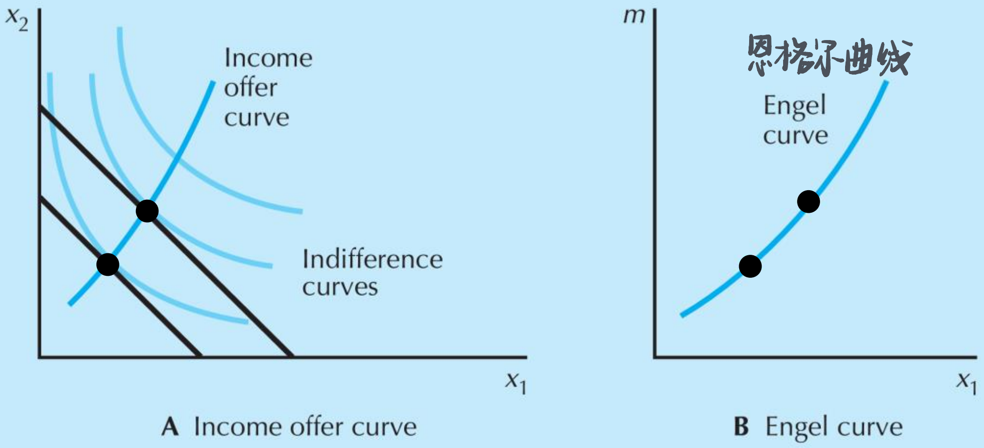
Perfect substitutes. Price offer curve (A) and demand curve (B) in the case of perfect substitutes.

Income Changes

How does $x_1{}^*(p_1, p_2, m)$ change as m changes, holding p_1 and p_2 constant?

Income Changes: Normal Goods

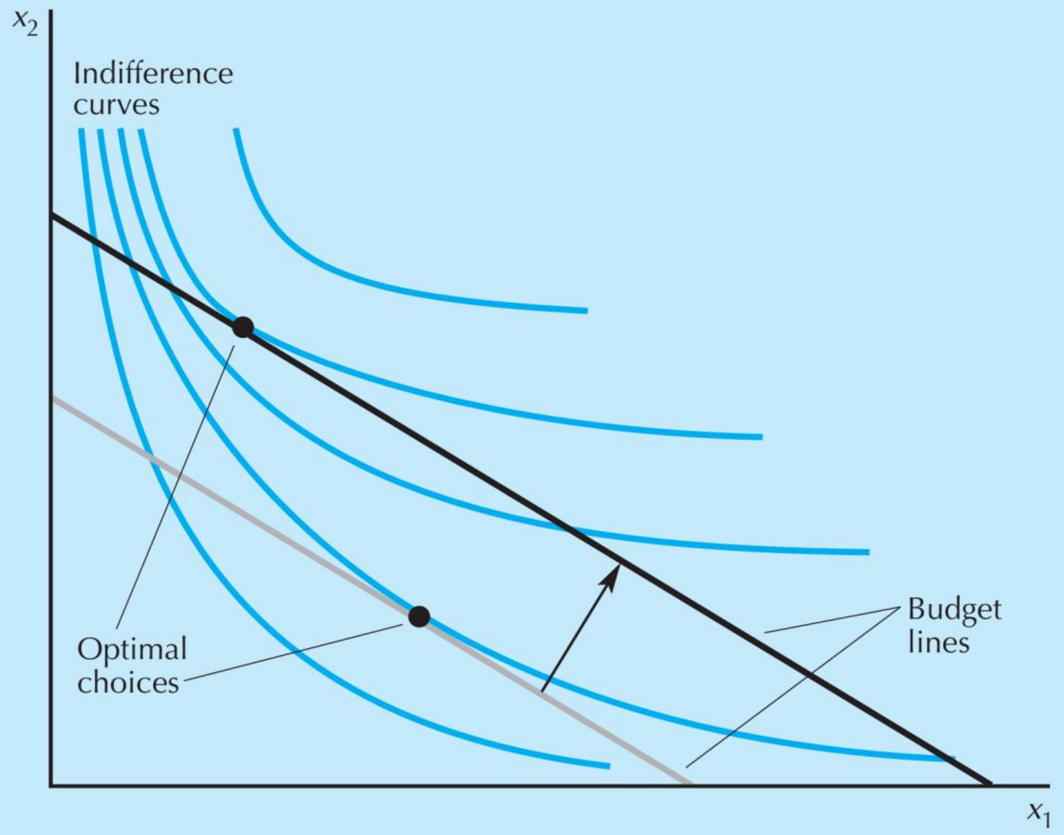
How does $x_1^*(p_1, p_2, m)$ change as m changes, holding p_1 and p_2 constant?



How demand changes as income changes. The income offer curve (or income expansion path) shown in panel A depicts the optimal choice at different levels of income and constant prices. When we plot the optimal choice of good 1 against income, m, we get the Engel curve, depicted in panel B.

低端品/劣等品.

Income Changes : Inferior Good 1



An inferior good. Good 1 is an inferior good, which means that the demand for it decreases when income increases.

Income Effects

A good for which quantity demanded rises with income is called normal.

—Normal goods include: luxury goods, neutral goods, and necessities.

—A normal good's Engel curve is always positively sloped ($\frac{\partial x_1}{\partial m} > 0$).

—Luxury goods $\frac{\partial^2 x_1}{\partial^2 m} > 0$

—Necessities $\frac{\partial^2 x_1}{\partial^2 m} < 0$

—Neutral goods $\frac{\partial^2 x_1}{\partial^2 m} = 0$

A good for which quantity demanded decreases with income is called inferior.

—An inferior good's Engel curve is always negatively sloped ($\frac{\partial x_1}{\partial m} < 0$).

Income Changes : Cobb-Douglas 中性商品

What does an income-offer curve look like for Cobb-Douglas utility function?

The ordinary demand functions for good 1 and good 2 are

$$x_1^*(p_1, p_2, m) = \frac{\alpha}{\alpha + \beta} \frac{m}{p_1}; \; x_2^*(p_1, p_2, m) = \frac{\beta}{\alpha + \beta} \frac{m}{p_2}$$

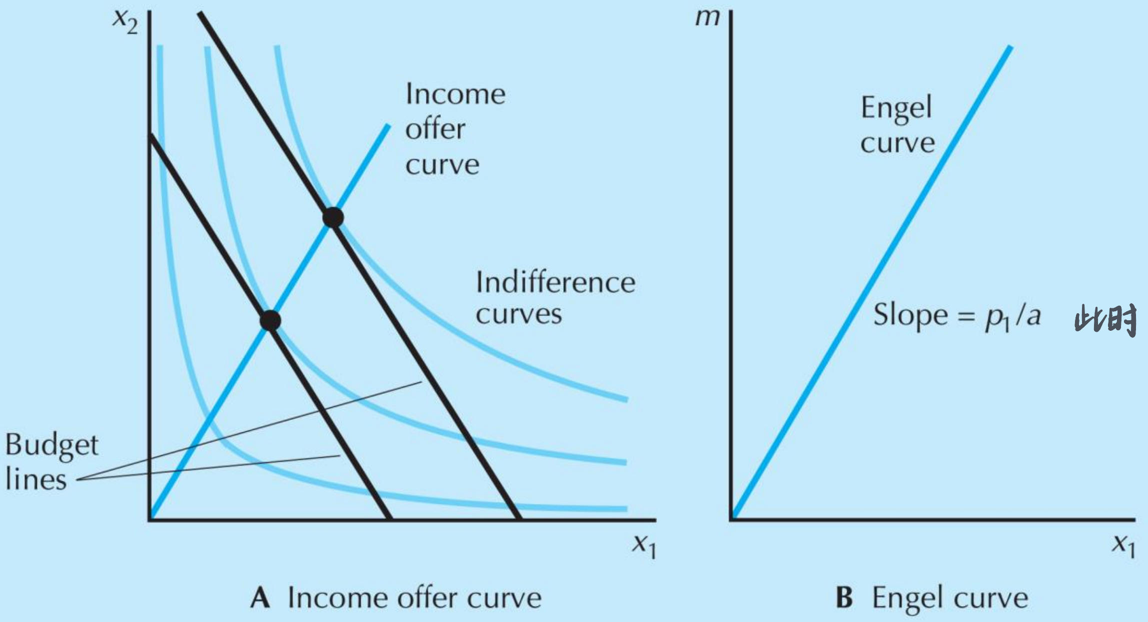
Rearrange to isolate m, we have

$$m = \frac{(\alpha+\beta)p_1}{\alpha} x_1^* \quad \text{Engel curve for good 1}$$

$$m = \frac{(\alpha+\beta)p_2}{\beta} x_2^* \quad \text{Engel curve for good 2}$$

Income Changes : Cobb-Douglas

What does an income-offer curve look like for Cobb-Douglas utility function?



此时 $a = \frac{\alpha}{\alpha+\beta}$.

Cobb-Douglas. An income offer curve (A) and an Engel curve (B) for Cobb-Douglas utility.

Income Changes : Perfect Complements

What does an income-offer curve look like for perfect-complements utility function?

The ordinary demand functions for good 1 and good 2 are

$$x_1^*(p_1, p_2, m) = x_2^*(p_1, p_2, m) = \frac{m}{p_1 + p_2}$$

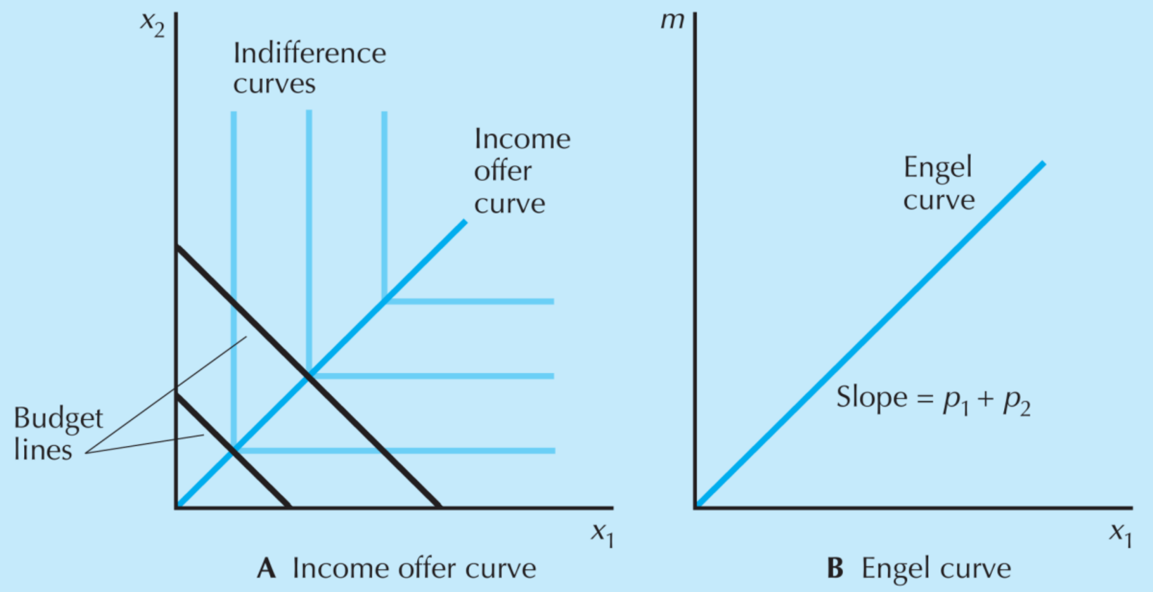
Rearrange to isolate m, we have

$$m = (p_1 + p_2)x_1^* \quad \text{Engel curve for good 1}$$

$$m = (p_1 + p_2)x_2^* \quad \text{Engel curve for good 2}$$

Income Changes : Perfect Complements

What does an income-offer curve look like for perfect-complements utility function?



Perfect complements. The income offer curve (A) and an Engel curve (B) in the case of perfect complements.

Income Changes : Perfect Substitutes

What does an income-offer curve look like for perfect-complements utility function?

The ordinary demand functions for good 1 and good 2 are

$$x_1{}^*(p_1, p_2, m) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ m/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

and

$$x_2{}^*(p_1, p_2, m) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ m/p_2 & , \text{if } p_1 < p_2 \end{cases}$$

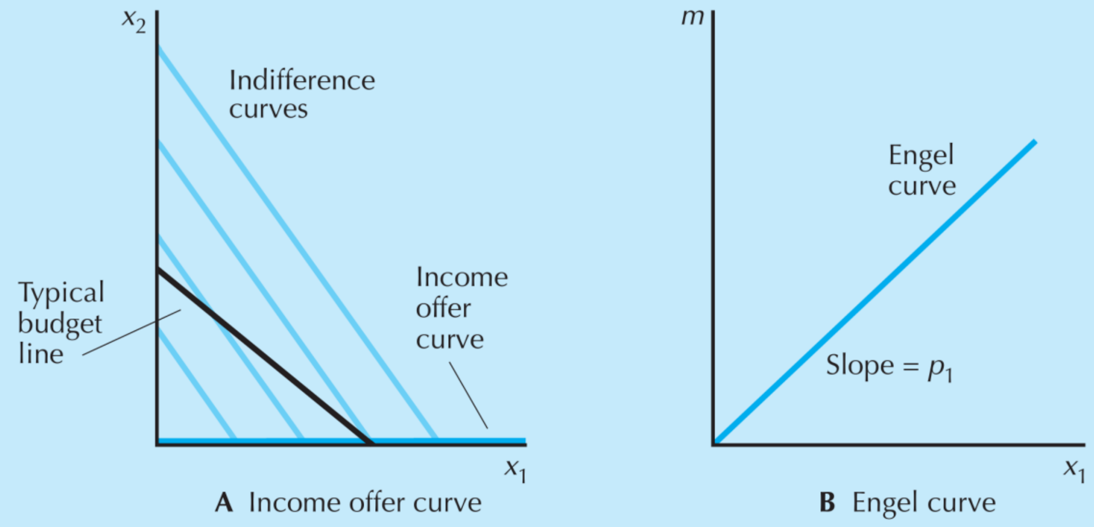
Rearrange to isolate m, we have

$m = p_1 x_1{}^*$ Engel curve for good 1

$m = p_2 x_2{}^*$ Engel curve for good 2

Income Changes : Perfect Substitutes

What does an income-offer curve look like for perfect-complements utility function?

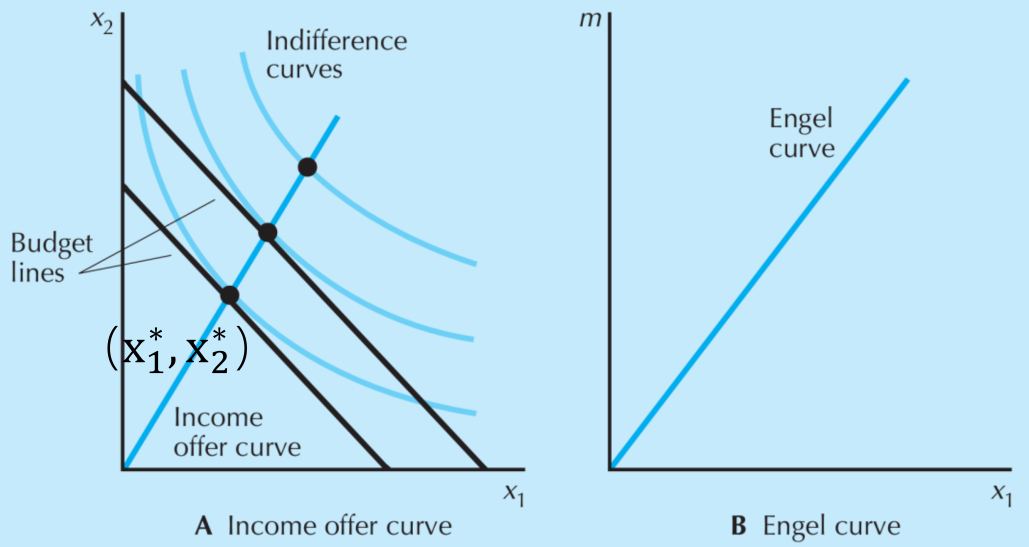


Perfect substitutes. The income offer curve (A) and an Engel curve (B) in the case of perfect substitutes.

Income Changes : Homothetic Preferences

相似偏好

What does an income-offer curve look like for homothetic preferences?



Homothetic preferences. An income offer curve (A) and an Engel curve (B) in the case of homothetic preferences.

Income Changes : Non-Homothetic Preferences

Quasilinear preferences are not homothetic.

For example,

$$u(x_1, x_2)=v(x_1) + x_2=\sqrt{x_1} + x_2$$

$MRS = \frac{\partial u / \partial x_1}{\partial u / \partial x_2} = \frac{\frac{1}{2\sqrt{x_1}}}{1} = \frac{p_1}{p_2}$

$p_1 x_1 + p_2 x_2 = m$

$\Rightarrow x_1^* = \left(\frac{p_2}{2p_1}\right)^2$

$x_2^* = \frac{m}{p_2} - \frac{p_2}{4p_1}$ ($x_2^* \geq 0$).

$m \geq \frac{p_2^2}{4p_1}$.

if $m < \frac{p_2^2}{4p_1}$. $x_2^* = 0 \Rightarrow x_1^* = \frac{m}{p_1}$.

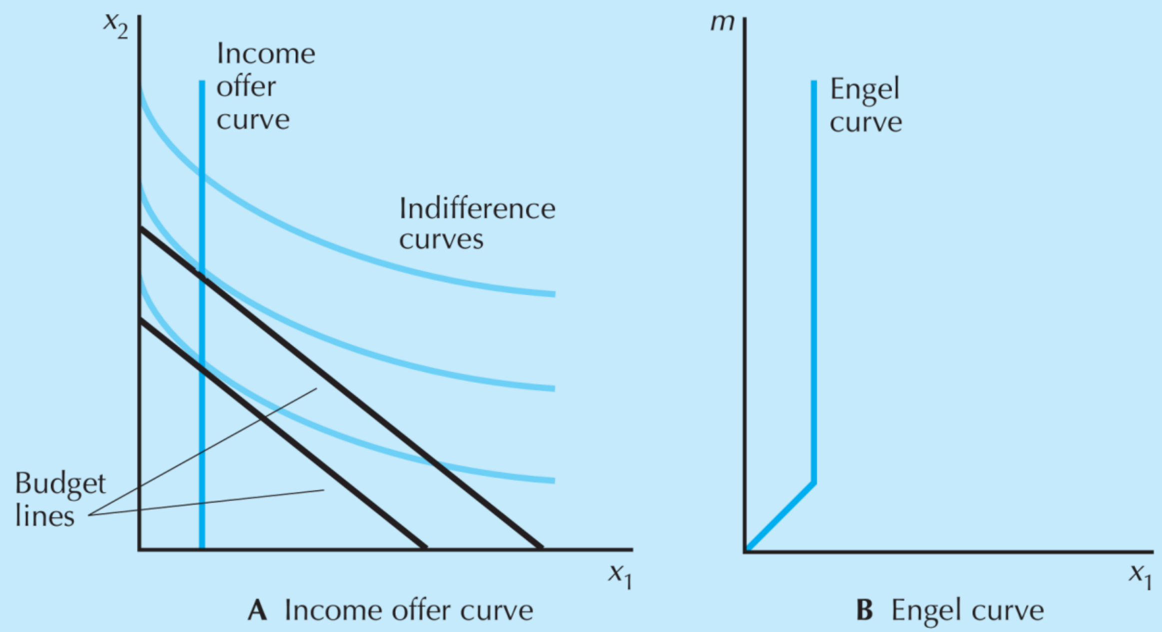
The ordinary demand functions for good 1 and good 2 are

$$x_1^*(p_1, p_2, m)=?$$

and

$$x_2^*(p_1, p_2, m)=?$$

Income Changes : Non-Homothetic Preferences



Quasilinear preferences. An income offer curve (A) and an Engel curve (B) with quasilinear preferences.

Summary

1. The consumer's demand function for a good will in general depend on the prices of all goods and income.

2. A normal good is one for which the demand increases when income increases. An inferior good is one for which the demand decreases when income increases

3. An ordinary good is one for which the demand decreases when its price increases. A Giffen good is one for which the demand increases when its price increases

4. If the demand for good 1 increases when the price of good 2 increases, then good 1 is a substitute for good 2. If the demand for good 1 decreases in this situation, then it is a complement for good 2